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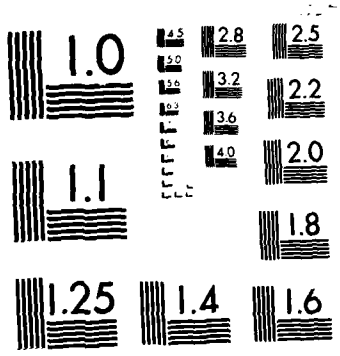

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LUMPED GEOPOTENTIAL HARMONICS OF ORDER 14, FROM THE ORBIT OF 1967-11G

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D. G. King-Hele

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LUMPED GEOPOTENTIAL HARMONICS OF ORDER 14, FROM THE ORBIT OF 1967-11G

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D. G. King-Hele

SUMMARY

The satellite 1967-11G, which had an orbital inclination of 40°, passed through 14th order resonance with the Earth's gravitational field in 1974. The changes in its orbital inclination at resonance have been analysed to obtain values for four lumped 14th-order harmonics in the geopotential, with accuracies equivalent to about 5 cm in geoid height. Analysis of the eccentricity was also attempted, but did not yield useful results.

As no previous satellite analysed at 14th-order resonance has had an inclination near 40°, the results have proved to be valuable in determining individual 14th-order harmonics in the geopotential.

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1 INTRODUCTION

A satellite is resonant with respect to the Earth's gravitational field when its track over the Earth repeats after a number of revolutions, perhaps 14 or 15. The analysis of such orbits is now well established as an accurate method for determining geopotential harmonics of a particular order, such as 15th order[1]. The geopotential is expressed as a double infinite series of tesseral harmonics of order m and degree $\ell(\geqslant m)$, and an orbit experiencing mth-order resonance is sensitive to harmonics of order m, and to a lesser extent those of order $2m$. Analysing a resonant satellite yields values of 'lumped' harmonics, which are linear sums of individual harmonic coefficients of order m. If a number of these mth-order resonant orbits at different inclinations are analysed, a number of values of lumped harmonics are obtained, and the resulting equations can be solved for individual harmonic coefficients.

Harmonics of order 14 are derived by analysing orbits which experience 14th-order resonance, *ie* the satellite's ground track over the Earth repeats every 14 revolutions. The orbits most suitable for analysis are those which contract very slowly under the influence of air drag and therefore remain close to 14th-order resonance for several years. Such orbits arise by chance from time to time, but the variety of inclination so far available has been inadequate. In a previous determination of individual 14th-order harmonics from resonance analysis[2], good results were obtained only for inclinations of 50°, 67°, 70°, 81° and 87°. However, orbits at such high inclinations are affected primarily by harmonics of degree less than 20, and lower-inclination orbits are essential if the evaluation of high-degree harmonics is to be improved.

In 1975 we asked for US Navy orbital elements for the satellite 1967-11G, inclination 40°, which passed through 14th-order resonance in 1974. (It was a fragment from the French satellite Diademe 1.) These orbital elements were not analysed at the time of the previous determination[2] of 14th-order harmonics, because 1967-11G passed through resonance quite rapidly and the orbital elements were at rather wide time intervals. Disappointingly, no subsequent satellite with an orbital inclination near 40° has passed more slowly through 14th-order resonance; so the orbital inclination and eccentricity of 1967-11G are now analysed to obtain values of lumped harmonics for use in a new evaluation of individual 14th-order harmonics. As expected, the analysis was quite difficult and not fully satisfactory: however, the results have proved to be good enough to give better-defined values for individual 14th-order harmonics of high degree[3].

2 THEORY

In this section the standard theory is repeated, so as to define the lumped coefficients $\bar{C}_m^{q,k}$ and $\bar{S}_m^{q,k}$. The longitude-dependent part of the geopotential can be written[4] in normalised form at an exterior point (r, θ, λ) as

$$\frac{\mu}{r} \sum_{\ell=2}^{\infty} \sum_{m=1}^{\ell} \left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m} (\cos \theta) \left\{ \bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda \right\} N_{\ell m} \quad , \qquad (1)$$

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where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), μ is the gravitational constant for the Earth ($398600\ km^3/s^2$), R is the Earth's equatorial radius (6378.1 km), $P_\ell^m(\cos\theta)$ is the associated Legendre function of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients, of which those of order $m = 14$ and 28 are relevant here. The normalizing factor $N_{\ell m}$ is given by[4]

$$N_{\ell m}^2 = \frac{2(2\ell + 1)(\ell - m)!}{(\ell + m)!} \quad . \tag{2}$$

The rate of change of inclination i caused by a relevant pair of coefficients, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, near $\beta{:}\alpha$ resonance may be written[2]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{\ell} \bar{F}_{\ell mp} G_{\ell pq}(k\cos i - m)\Re\left[j^{\ell-m+1}(\bar{C}_{\ell m} - j\bar{S}_{\ell m})\exp\{j(\gamma\Phi - q\omega)\}\right] \quad , \tag{3}$$

where $\bar{F}_{\ell mp}$ is Allan's normalized inclination function[5], $G_{\ell pq}$ is a function of eccentricity e for which explicit forms have been derived by Gooding[6], $\Re$ denotes 'real part of' and $j = \sqrt{-1}$. The resonance angle Φ is defined by the equation

$$\Phi = \alpha(\omega + M) + \beta(\Omega - \nu) \quad , \tag{4}$$

where ω is the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν the sidereal angle. The indices γ, q, k and p in equation (3) are integers, with γ taking the values 1, 2, 3 .... and q the values 0, ±1, ±2, ...; the equations linking ℓ, m, k and p are: $m = \gamma\beta$; $k = \gamma\alpha - q$; $2p = \ell - k$. In equation (3) there is a factor $(1 - e^2)^{-\frac{1}{2}}$ on the right-hand side, which is taken as 1.

Here $\beta = 14$ and $\alpha = 1$, and the m-suffix of a relevant $(\bar{C}_{\ell m}, \bar{S}_{\ell m})$ pair is decided by the choice of γ. The values of ℓ to be taken must be such that $\ell \geq m$ and $(\ell - k)$ is even. The successive coefficients which arise (for given γ and q) may be grouped into a lumped harmonic, written as[2]

$$\bar{C}_m^{q,k} = \sum_{\ell} Q_\ell^{q,k}\bar{C}_{\ell m} \quad , \qquad \bar{S}_m^{q,k} = \sum_{\ell} Q_\ell^{q,k}\bar{S}_{\ell m} \quad , \tag{5}$$

where ℓ increases in steps of two from its minimum permissible value ℓ_0, and the Q_ℓ are constant coefficients, with $Q_{\ell 0} = 1$. The values of the Q_ℓ can be obtained from equation (3), and R.H. Gooding has written a computer program PROF for their evaluation.

The rate of change of eccentricity e caused by the (ℓ, m) harmonic near $\beta{:}\alpha$ resonance can be written[2]

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{\ell} \bar{F}_{\ell mp} G_{\ell pq} \left\{\frac{q - \frac{1}{2}(k+q)e^2}{e}\right\} \Re\left[j^{\ell-m+1}(\bar{C}_{\ell m} - jS_{\ell m}) \exp j(\gamma\Phi - q\omega)\right] \quad . \qquad \ldots\ldots(6)$$

Again there is a factor $(1 - e^2)^{-\frac{1}{2}}$ on the right-hand side, which is taken as 1.

The relative importance of the many possible terms arising in equations (3) and (6) is indicated in section 3.

3 PRECONCEPTIONS OF THE TERMS IMPORTANT FOR 1967-11G

3.1 Inclination

The main terms contributing to di/dt in equation (3) are listed in Table 1, with the factor $n(R/a)^{15}$ (14 cosec i - cot i) taken out. For 1967-11G at resonance this factor has the value 301 radians/day.

Table 1

The main terms in the equation for di/dt

q \ γ	1	2
0	$\bar{F}_{15,14,7}\left\{\bar{S}^{0,1}_{14} \sin\Phi + \bar{C}^{0,1}_{14} \cos\Phi\right\}$	$2\left(\frac{R}{a}\right)^{13} \bar{F}_{28,28,13}\left\{\bar{C}^{0,2}_{28} \sin 2\Phi - \bar{S}^{0,2}_{28} \cos 2\Phi\right\}$
1	$\frac{15}{2} e \left(\frac{a}{R}\right) \frac{14}{14 - \cos i} \bar{F}_{14,14,7}$ $\left\{\bar{C}^{1,0}_{14} \sin(\Phi - \omega) - \bar{S}^{1,0}_{14} \cos(\Phi - \omega)\right\}$	$16e \left(\frac{R}{a}\right)^{14} \frac{28 - \cos i}{14 - \cos i} \bar{F}_{29,28,14}$ $\left\{\bar{S}^{1,1}_{28} \sin(2\Phi - \omega) + \bar{C}^{1,1}_{28} \cos(2\Phi - \omega)\right\}$
-1	$\frac{11}{2} e \left(\frac{a}{R}\right) \frac{14 - 2\cos i}{14 - \cos i} \bar{F}_{14,14,6}$ $\left\{\bar{C}^{-1,2}_{14} \sin(\Phi + \omega) - \bar{S}^{-1,2}_{14} \cos(\Phi + \omega)\right\}$	$12e \left(\frac{R}{a}\right)^{14} \frac{28 - 3\cos i}{14 - \cos i} \bar{F}_{29,28,13}$ $\left\{\bar{S}^{-1,3}_{28} \sin(2\Phi + \omega) + \bar{C}^{-1,3}_{28} \cos(2\Phi + \omega)\right\}$
2	$\frac{239}{8} e^2 \frac{14 + \cos i}{14 - \cos i} \bar{F}_{15,14,8}$ $\left\{\bar{S}^{2,-1}_{14} \sin(\Phi - 2\omega) + \bar{C}^{2,-1}_{14} \cos(\Phi - 2\omega)\right\}$	–
-2	$\frac{133}{8} e^2 \frac{14 - 3\cos i}{14 - \cos i} \bar{F}_{15,14,6}$ $\left\{\bar{S}^{-2,3}_{14} \sin(\Phi + 2\omega) + \bar{C}^{-2,3}_{14} \cos(\Phi + 2\omega)\right\}$	–

The orders of magnitude of the lumped coefficients $(\bar{C}, \bar{S})^{q,k}_{14}$ in Table 1 can be assessed very approximately by assuming[4] that $\bar{C}_{\ell,m}$ is of order $10^{-5}/\ell^2$ and taking $\bar{C}^{q,k}_m$ to be of order

$$\left\{\sum_{\ell} \left(Q^{q,k}_{\ell}\ 10^{-5}/\ell^2\right)^2\right\}^{\frac{1}{2}} \quad ;$$

similarly for $\bar{S}^{q,k}_m$. Sometimes, of course, the real values of the lumped coefficients may be small by chance, and this method will overestimate their important. Table 2 gives the values calculated in this way, all $\times\ 10^{-9}$

Table 2

Orders of magnitude of terms in Table 1, for 1967-11G

q \ γ	1	2
0	7.1	0.25
1	2.4	0.2
-1	2.7	0.2
2	0.5	-
-2	0.5	-

From Table 2 it appears that only the terms in $(\gamma, q) = (1, 0)$, $(1, 1)$ and $(1, -1)$ are likely to be of importance.

3.2 Eccentricity

The main terms contributing to de/dt in equation (6) are listed in Table 3, with the factor $n(R/a)^{15}$ taken out. For 1967-11G at resonance this factor has the value $14.6\ \text{day}^{-1}$.

Table 3

The main terms in the equation for de/dt

q \ γ	1	2
0	$\frac{e}{2}\bar{F}_{15,14,7}\left\{\bar{S}_{14}^{0,1}\sin\Phi+\bar{C}_{14}^{0,1}\cos\Phi\right\}$	–
1	$-\frac{15}{2}\left(\frac{a}{R}\right)\bar{F}_{14,14,7}\left\{\bar{C}_{14}^{1,0}\sin(\Phi-\omega)-\bar{S}_{14}^{1,0}\cos(\Phi-\omega)\right\}$	$-16\left(\frac{R}{a}\right)^{14}\bar{F}_{29,28,14}\left\{\bar{S}_{28}^{1,1}\sin(2\Phi-\omega)+\bar{C}_{28}^{1,1}\cos(2\Phi-\omega)\right\}$
-1	$\frac{11}{2}\left(\frac{a}{R}\right)\bar{F}_{14,14,6}\left\{\bar{C}_{14}^{-1,2}\sin(\Phi+\omega)-\bar{S}_{14}^{-1,2}\cos(\Phi+\omega)\right\}$	$12\left(\frac{R}{a}\right)^{14}\bar{F}_{29,28,13}\left\{\bar{S}_{28}^{-1,3}\sin(2\Phi+\omega)+\bar{C}_{28}^{-1,3}\cos(2\Phi+\omega)\right\}$
2	$-\frac{239e}{4}\bar{F}_{15,14,8}\left\{\bar{S}_{14}^{2,-1}\sin(\Phi-2\omega)+\bar{C}_{14}^{2,-1}\cos(\Phi-2\omega)\right\}$	–
-2	$\frac{133e}{4}\bar{F}_{15,14,6}\left\{\bar{S}_{14}^{-2,3}\sin(\Phi+2\omega)+\bar{C}_{14}^{-2,3}\cos(\Phi+2\omega)\right\}$	–

The orders of magnitude of the terms, calculated in the same way as for di/dt , are given in Table 4.

Table 4

Orders of magnitude of terms in Table 3, for 1967-11G

q \ γ	1	2
0	0.1	-
1	63	2.3
-1	76	2.8
2	24	-
-2	28	-

From Table 4 it appears that, for e , the terms in $(\gamma, q) = (1, 1)$ and $(1, -1)$ will be the most important, though terms in $(\gamma, q) = (1, 2)$ and $(1, -2)$ may also be significant.

An alternative method for evaluating the orders of magnitude of the terms would be to take the larger of the $\bar{C}_{\ell m}$ or $\bar{S}_{\ell m}$ in a particular Earth model. Unfortunately, with 1967-11G, the largest Q_ℓ are for $24 < \ell < 61$, where the values in existing Earth models are not considered very reliable.

4 THE PASSAGE THROUGH RESONANCE

At the time of its passage through resonance, 1967-11G had a perigee height near 540 km, an apogee height near 1100 km and an orbital period of 101.3 minutes.

There were 59 sets of US Navy orbital elements available, at dates between 19 April 1974 (modified Julian day 42156) and 10 January 1976 (MJD 42787). During this time the value of $\dot{\Phi}$, given by equation (4), changed from -3.99 deg/day to +8.68 deg/day, with exact resonance ($\dot{\Phi} = 0$) on 9 September 1974 (MJD 42299). Fig 1 gives the variation of $\dot{\Phi}$ as a broken line, and it can be seen that its rate of change became much slower in 1975, because the air density was then lower than in 1974, as a result of lower solar activity. The unbroken line in Fig 1 shows the variation of Φ over the same time interval.

Fig 1 shows that the time interval covered by the orbital data is far from symmetrical about the exact resonance, and this has several implications for the analysis. First, the terms in $(\gamma, q) = (1, -1)$, that is the terms in $\begin{matrix}\cos\\ \sin\end{matrix} (\Phi + \omega)$, are likely to have an appreciable effect only near the time when $(\Phi + \omega)$ is nearly constant, that is when $\dot{\Phi} \simeq - \dot{\omega}$. But $\dot{\omega} = 6.3$ deg/day for 1967-11G, so the main effects of the $(\gamma, q) = (1, -1)$ terms occur when $\dot{\Phi} = - 6.3$ deg/day - well before the start of the orbital data. Thus it is likely that the coefficients of the $(\gamma, q) = (1, -1)$ terms will prove to be indeterminate; and this is just what happens. The same applies, *a fortiori*, with the $(\gamma, q) = (1, -2)$ terms.

So the terms to be considered in the fittings are $(\gamma, q) = (1,0), (1, 1)$ and possibly $(2, 0)$ for inclination, and $(\gamma, q) = (1,1)$ and $(1,2)$ for eccentricity.

5 ANALYSIS OF INCLINATION

5.1 Treatment of the data

The US Navy orbital elements were first converted, with the ELTRAN2 computer program, to midnight epochs in the five-card format of the RAE PROP orbital elements. The lunisolar and zonal harmonic pertubations to i were calculated with the aid of the program PROD[7], with a one-day integration interval and restarts at intervals of about 40 days. The perturbations indicated by PROD, which never exceeded 0.0042°, were then subtracted from the raw values. The maximum di/dt due to solar radiation pressure perturbations was estimated as 3×10^{-6} deg/day, which is negligible since di/dt due to resonance is 2×10^{-4} deg/day. Perturbations due to tides were also ignored.

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Fig 2 shows the values of inclination cleared of perturbations. Initially all the values of i were assumed to have errors of 0.003°.

5.2 The fittings and their results

These values were then fitted with an integrated form of the theoretical equation (3), with the aid of the THROE computer program[8], for various pairs of values of (γ, q), and assuming an atmospheric rotation rate $\Lambda = 0.9$ rev/day. The first run was with $(\gamma, q) = (1, 0)$, since Table 2 indicates that these should be the dominant terms. The results were quite good, with the measure of fit ε having the value 1.5 (we define ε^2 as the sum of the squares of the weighted residuals, divided by the number of degrees of freedom). However, it was obvious that some of the values were ill-fitting and would remain so - particularly the high values near MJD 42600 visible in Fig 2. The accuracy of a number of these values was relaxed by a factor of 2, and ε then decreased to 0.96. Next the $(\gamma, q) = (1, 1)$ terms were added and slightly improved the fit ($\varepsilon = 0.95$) without much altering the values for the $(\gamma, q) = (1, 0)$ terms. Further relaxations in accuracy were then made, by doubling the sd of any values with residual exceeding 2ε : this reduced ε to 0.80. Finally the $(\gamma, q) = (2, 0)$ terms were added: this reduced ε to 0.75 and, although the values for the (2, 0) terms were unlikely to be reliable, the improvement in fitting seemed worthwhile, so the (2, 0) terms were retained. Some further relaxations and restorations in accuracy were now needed, and in the final run the assumed sd of each value is as shown in Fig 2. The value of ε was 0.68, and the values of the lumped harmonics which emerged are as follows:

$$10^9 \bar{C}^{0,1}_{14} = -65 \pm 163 , \quad 10^9 \bar{S}^{0,1}_{14} = -55 \pm 164 , \tag{7}$$

$$10^6 \bar{C}^{1,0}_{14} = 1.3 \pm 1.6 , \quad 10^6 \bar{S}^{1,0}_{14} = 2.5 \pm 1.3 , \tag{8}$$

$$10^3 \bar{C}^{0,2}_{28} = 1.4 \pm 0.4 , \quad 10^3 \bar{S}^{0,2}_{28} = -1.1 \pm 0.5 . \tag{9}$$

The standard deviations in equations (7) to (9) are rather meaningless as they stand, because of the large Q values of 1967-11G. To assess the significance of the standard deviations, it is worth trying to give an indication of the error in geoid height to which they correspond; and this can perhaps best be done by dividing each sd by the largest of the relevant Q factors and then multiplying by the Earth radius (6378 km). The approximate geoid height accuracies for the three pairs of standard deviations in equations (7) to (9) are then found to be 7 cm, 5 cm, and 70 cm respectively. The 28th-order harmonics have too large an error to be of interest, and will be ignored; but the first two pairs have accuracies similar to those achieved with other orbits analysed at 14th-order resonance.

On the basis of the $10^{-5}/\ell^2$ rule, the values of the $(\bar{C}, \bar{S})^{0,1}_{14}$ would be 600×10^{-9}, so the values (7) are unusually small - and that is why they are smaller than their sd. The $10^{-5}/\ell^2$ rule gives $(\bar{C}, \bar{S})^{1,0}_{14}$ as of order 6×10^{-6}; so the values

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(8) are of about the expected magnitude. The $(C, S)_{28}^{0,2}$ terms should be of order 0.1×10^{-3} on the $10^{-5}/\ell^2$ rule; probably, therefore, the values (9) are considerably too large.

5.3 Equations for the individual harmonic coefficients

Each of the lumped harmonics $(\bar{C}, \bar{S})_{14}^{0,1}$ and $(\bar{C}, \bar{S})_{14}^{1,0}$ can be expressed as a linear sum of individual harmonic coefficients, as indicated in equation (5). For 1967-11G the explicit forms of these equations are as follows

$$\bar{C}_{14}^{0,1}: \quad \bar{C}_{15,14} - 4.48\bar{C}_{17,14} + 10.39\bar{C}_{19,14} - 15.07\bar{C}_{21,14} + 13.78\bar{C}_{23,14} - 6.25\bar{C}_{25,14}$$
$$- 1.96\bar{C}_{27,14} + 4.91\bar{C}_{29,14} - 2.23\bar{C}_{31,14} - 1.38\bar{C}_{33,14} + 2.06\bar{C}_{35,14}$$
$$= (-65 \pm 163) \times 10^{-9} \quad . \tag{10}$$

$\bar{S}_{14}^{0,1}$: the equation is the same as (10) with C replaced by S and the numerical value on the right-hand side changed to $(-55 \pm 164) \times 10^{-9}$. (11)

$$\bar{C}_{14}^{1,0}: \quad \bar{C}_{14,14} - 10.6\bar{C}_{16,14} + 43.9\bar{C}_{18,14} - 106.8\bar{C}_{20,14} + 170.3\bar{C}_{22,14} - 178.5\bar{C}_{24,14}$$
$$+ 103.7\bar{C}_{26,14} + 8.6\bar{C}_{28,14} - 72.9\bar{C}_{30,14} + 51.1\bar{C}_{32,14} + 10.5\bar{C}_{34,14}$$
$$- 39.4\bar{C}_{36,14} = (1.3 \pm 1.6) \times 10^{-6} \quad . \tag{12}$$

$\bar{S}_{14}^{1,0}$: the equation is the same as (12) with C replaced by S and the numerical value on the right-hand side changed to $(2.5 \pm 1.3) \times 10^{-6}$. (13)

The series on the left-hand sides are terminated at degree 36 because it is unlikely that higher-degree terms can be successfully evaluated until we have much more data.

Equations (10) to (13) show that the individual coefficients of degree 19-24 are likely to make the largest contribution to the lumped harmonics.

6 ANALYSIS OF ECCENTRICITY

The prospects of successfully analysing the variation in eccentricity seemed poor from the outset, because there were such large changes in e due (presumably) to perturbations other than resonance. The values of e after correction for the effects of zonal harmonics and air drag still exhibited a large variation - from near 0.039 initially to more than 0.040. This corresponds to a change in perigee distance of more than 7 km, which is much greater than would be expected as a result of resonance. In the initial THROE run, the first 15 values of e were ill-fitting, and there seemed no harm in omitting them, because the resonant variation of eccentricity is 'centred' on the point where $\dot{\ell} = \dot{\omega}$, which is quite late in the data set, at MJD 42550 (see Fig 1).

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In subsequent shortened runs the early values still fitted badly: eventually, only 33 values of eccentricity were retained, and the fitting, with $(\gamma, q) = (1, 1)$, is shown in Fig 3. This gave

$$10^6\bar{C}^{1,0}_{14} = 4.0 \pm 1.7 , \qquad 10^6\bar{S}^{1,0}_{14} = 19.3 \pm 1.7 , \tag{14}$$

with $\varepsilon = 1.16$, after accuracies were relaxed by a factor of 2 for all values having residuals greater than 2ε . The density scale height was taken as 75 km.

The value of $\bar{S}^{1,0}_{14}$ expected from the $10^{-5}/\ell^2$ rule is 6×10^{-6} so the high value in equation (14) is suspect, and the likely culprit is solar radiation pressure. The largest contribution to the equation for the variation in e produced by solar radiation pressure[9] is from the term in $\sin(\Omega + \omega - L)$, where L is the solar longitude. For 1967-11G, the angle $(\Omega + \omega - L)$ increases at a rate of only 0.32° per day, from 120° at MJD 42479 to 220° at MJD 42787: consequently the variation in e due to solar radiation pressure closely mimics in form the curve of Fig 3, being sinusoidal with a maximum near MJD 42670. It is not possible to remove the srp effects reliably, because

a. the mass, size and shape of 1967-11G are unknown;

b. its decay rate is irregular, suggesting irregular variations in the effective cross-sectional area;

c. the srp cross-section is probably also variable, and may vary differently to the drag cross-section; and

d. the srp effect is probably larger than the resonance effect and mimics it in form.

The average decay rate of 1967-11G indicates a mass/area ratio of about 20 kg/m^2. Taking this value for the srp mass/area and using only the main term in the srp perturbation equation, we find the values of e after removal of srp effects are as shown in Fig 4. The THROE fitting, shown by the curve in Fig 4, now gives

$$10^6\bar{C}^{1,0}_{14} = -0.7 \pm 1.6 , \qquad 10^6\bar{S}^{1,0}_{14} = 7.5 \pm 1.6 , \tag{15}$$

with $\varepsilon = 1.10$. The values (15) are quite different to (14), and $\bar{S}^{1,0}_{14}$ could be reduced to a value consistent with (8) if the srp effect was increased by 25%, which is about the probable limit of error. We must conclude that the values (14) are quite spurious, because of the neglect of solar radiation pressure, while the values (15) need to be given larger sds - perhaps about ±4 - to take account of the possible errors in the removal of srp effects.

In Fig 4 there is an oscillation in unison with ω , so a fitting with $(\gamma, q) = (1, 1)$ and $(0, 1)$ was also made, and is shown in Fig 5. This gave

$$10^6\bar{C}^{1,0}_{14} = 0.3 \pm 1.2 , \qquad 10^6\bar{S}^{1,0}_{14} = 9.7 \pm 1.0 , \tag{16}$$

with $\varepsilon = 0.80$, after several of the relaxed sds were restored to the standard value. Though nominally more accurate than (15), the sds in (16) still need to be augmented to allow for the possible mis-estimation of the srp effects, and again values of about ±4 may be appropriate.

All in all, the values of equation (8) seem likely to be more reliable than (15) or (16).

7 CONCLUSIONS

The satellite 1967-11G passed through 14th-order resonance with the Earth's gravitational field in September 1974. Analysis of the changes in inclination between April 1974 and January 1976, as given by 59 US Navy orbits (Fig 2), has yielded values for four lumped harmonics of order 14, and the accuracy of the evaluation is equivalent to about 5 cm in geoid height: see equations (7) and (8). The eccentricity was also analysed, but the results are regarded as unreliable because of uncertainties in the evaluation of perturbations due to solar radiation pressure, which are larger than the resonance effects.

Among satellites previously analysed as they passed through 14th-order resonance, none has had an inclination between 34° and 48°, so the results from 1967-11G at 40° inclination provide new information which should be useful in determining individual harmonic coefficients of order 14 - particularly those of high degree. This expectation has proved valid[3].

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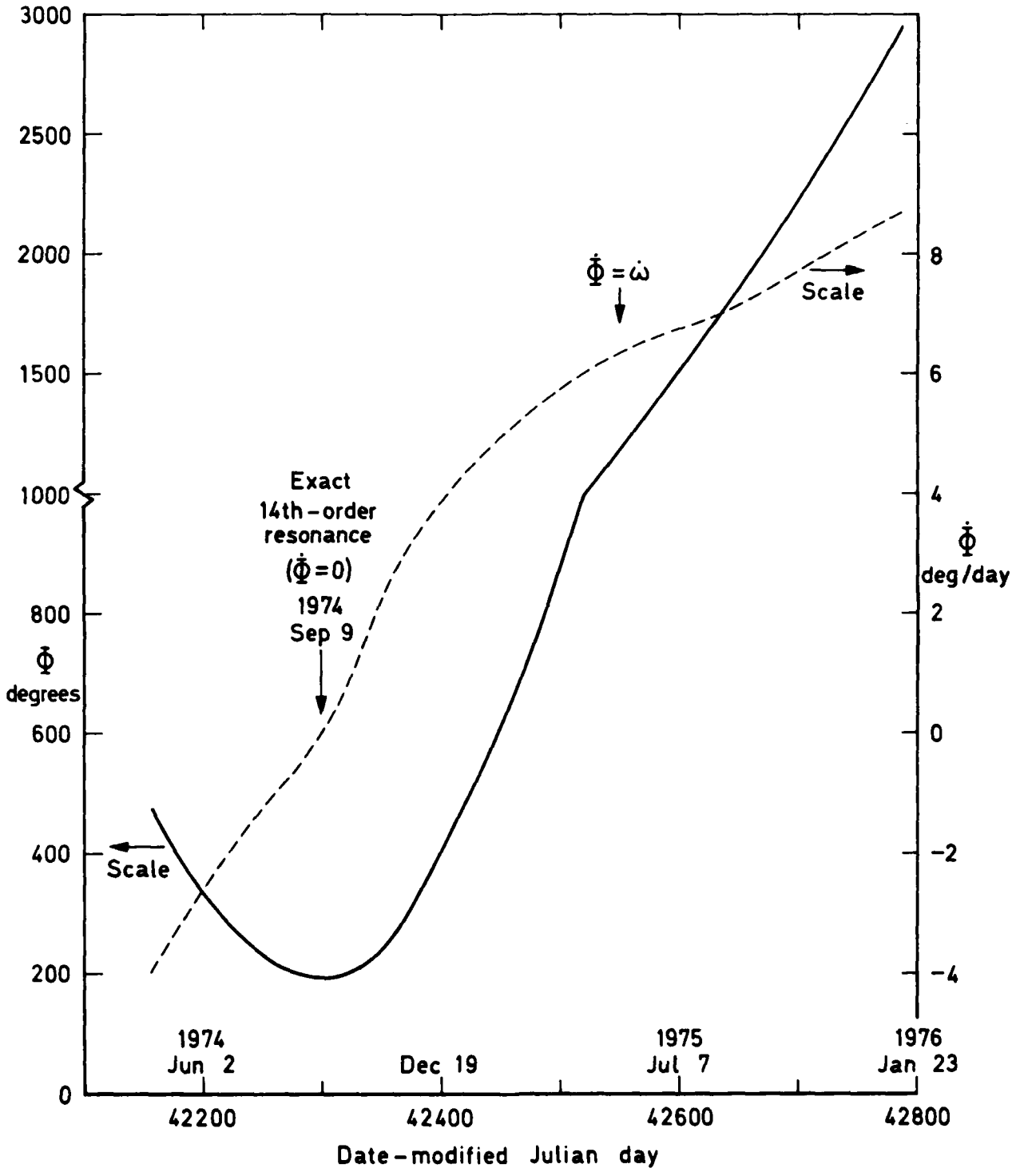

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Fig 1 Variation of Φ (full line) and $\dot{\Phi}$ (broken line)

Fig 2

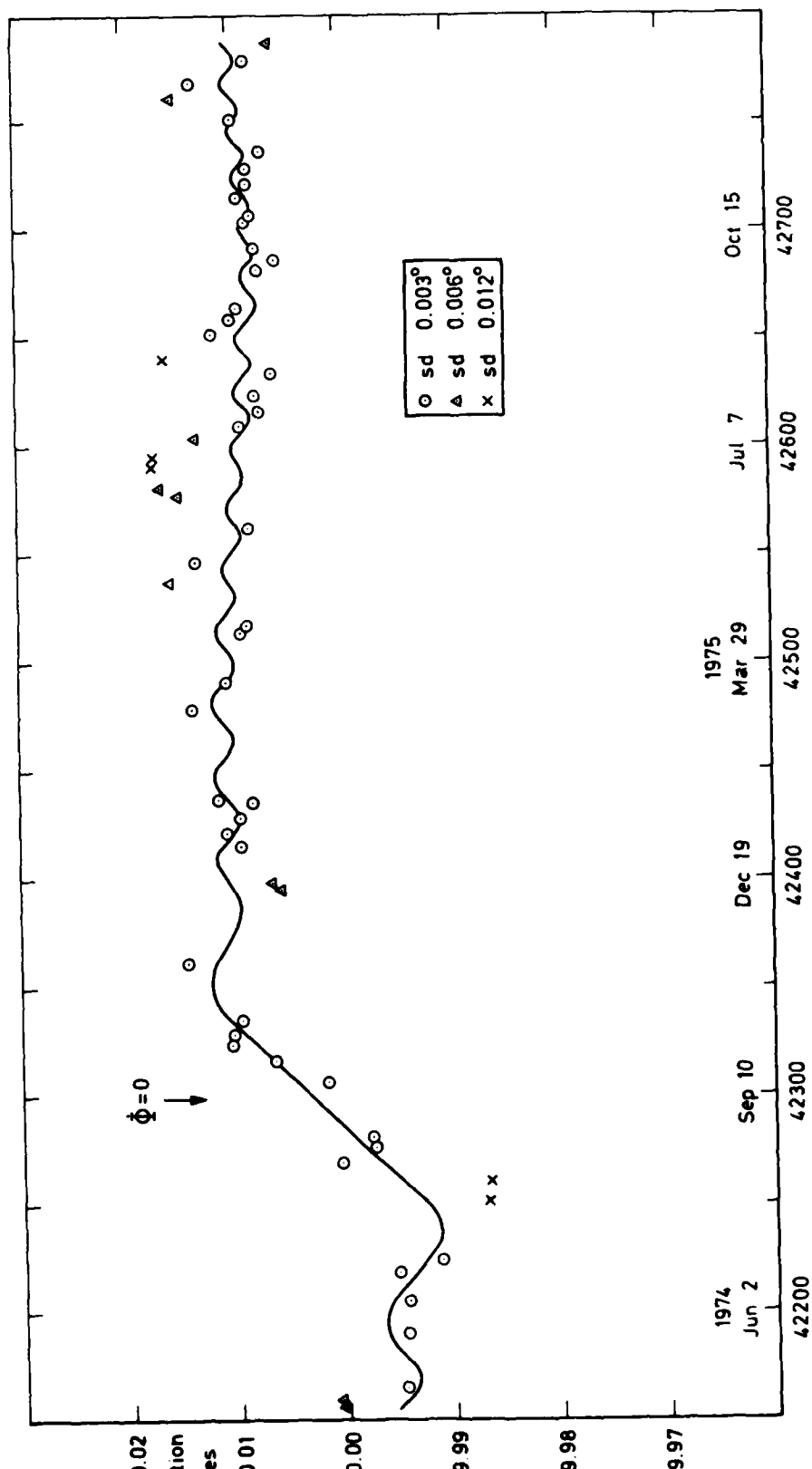

Fig 2 Values of inclination, after removal of perturbations, fitted with $(\gamma,q) = (1,0), (1,1)$ and $(2,0)$

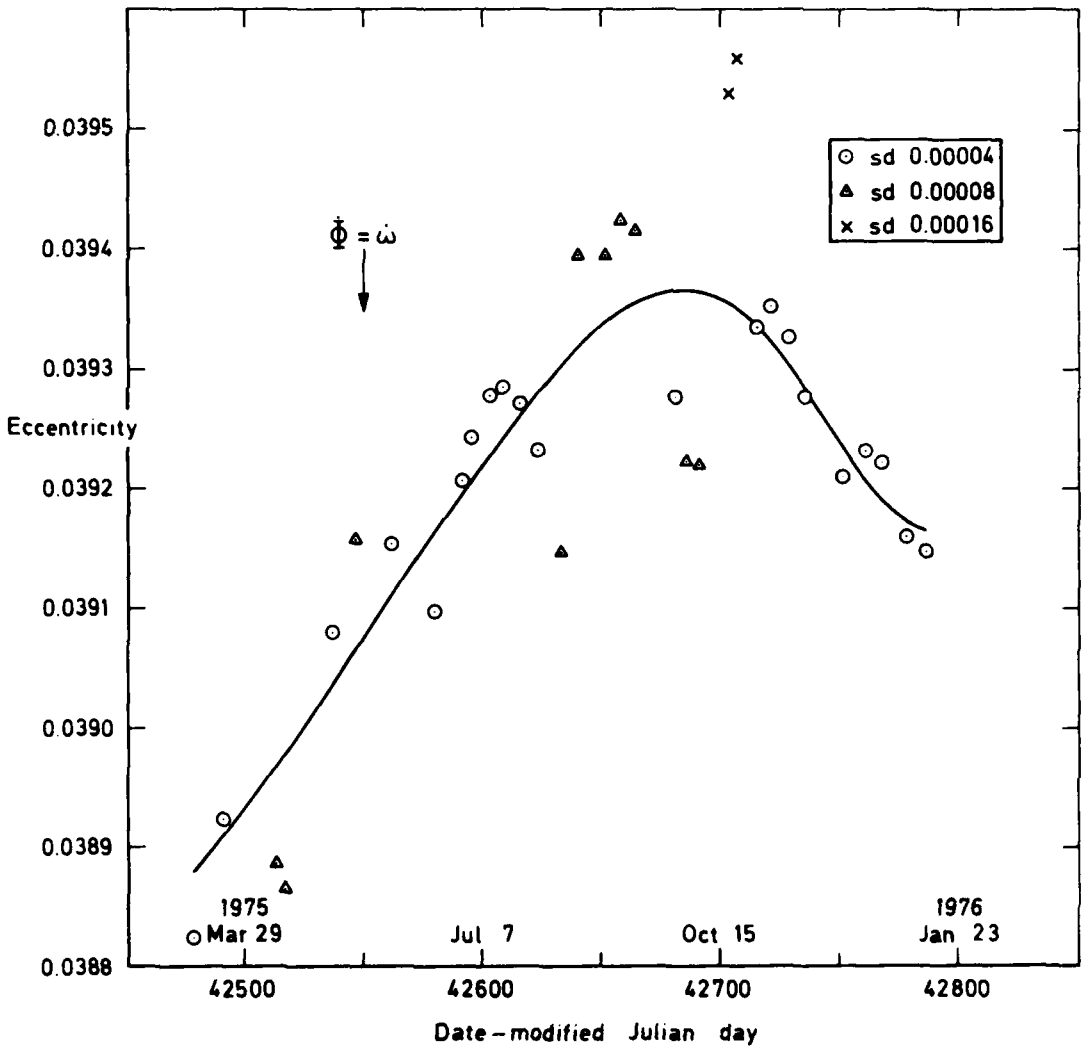

Fig 3 Values of eccentricity, after removal of air drag and zonal harmonic perturbations, fitted with $(\gamma,q) = (1,1)$

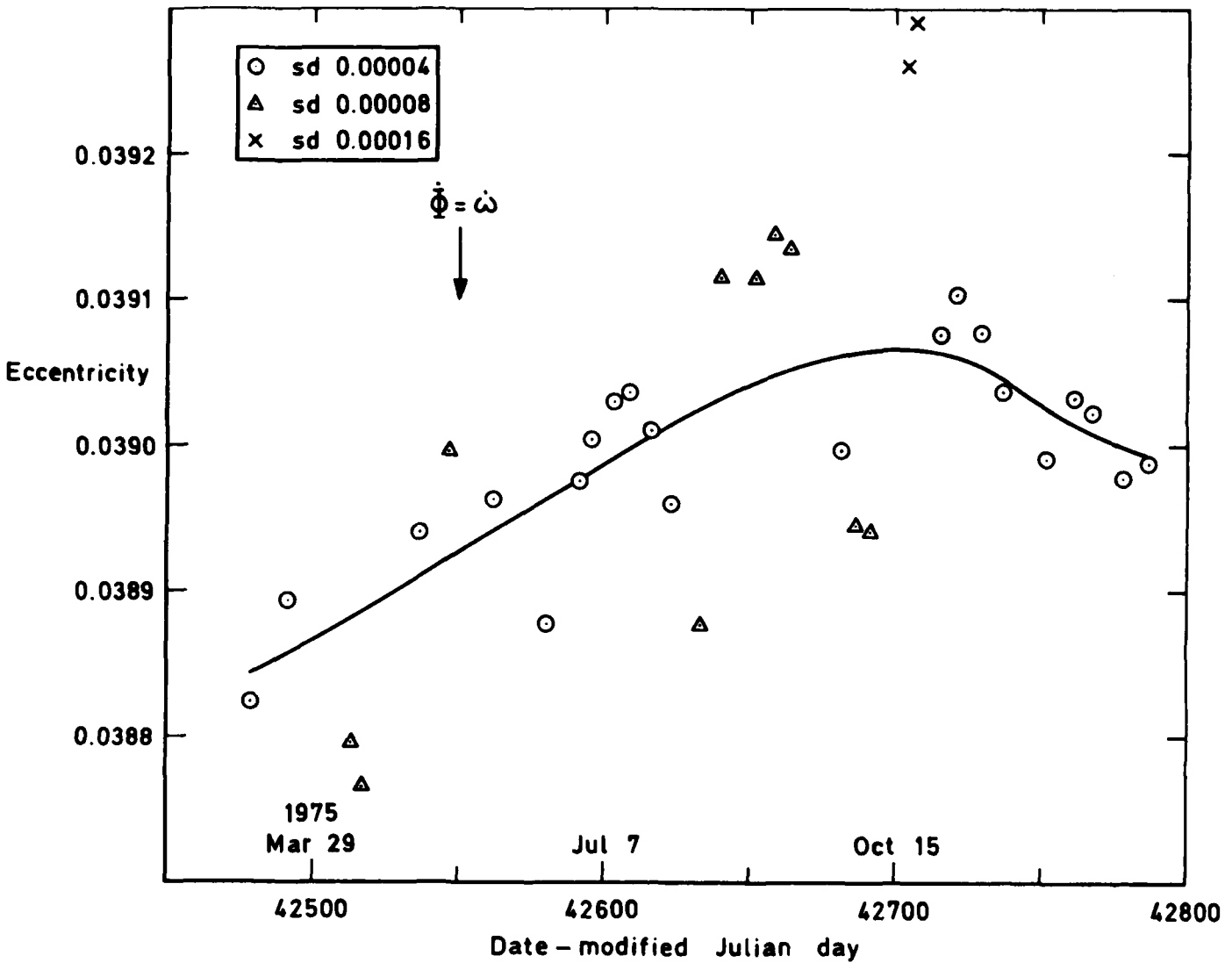

Fig 4 Values of eccentricity, after removal of air drag, zonal harmonic and srp perturbations, fitted with $(\gamma,q) = (1,1)$

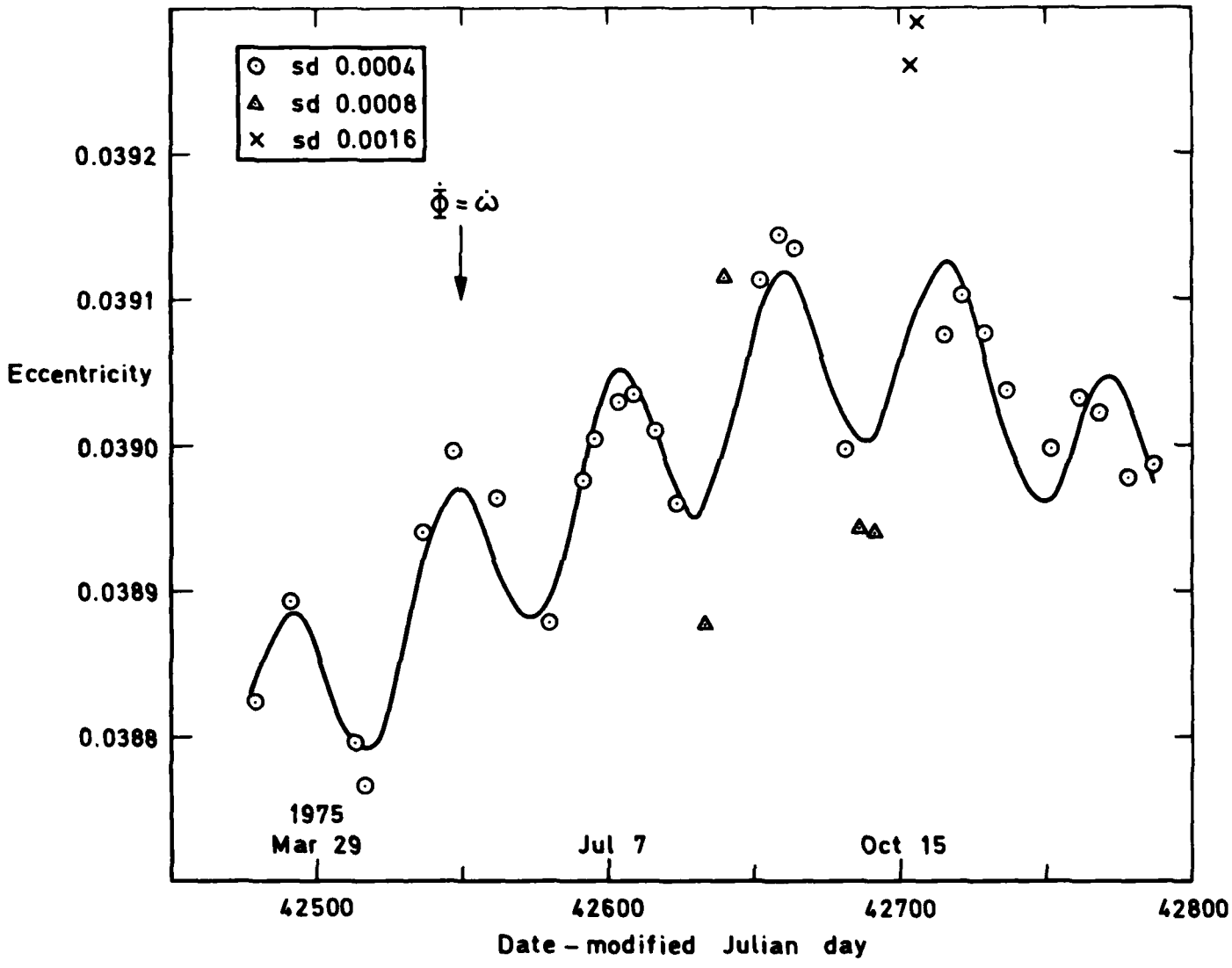

Fig 5 Values of eccentricity, after removal of air drag, zonal harmonic and srp perturbations, fitted with $(\gamma,q) = (1,1)$ and $(0,1)$

REPORT DOCUMENTATION PAGE

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17. Abstract

The satellite 1967-11G, which had an orbital inclination of 40°, passed through 14th-order resonance with the Earth's gravitational field in 1974. The changes in its orbital inclination at resonance have been analysed to obtain values for four lumped 14th-order harmonics in the geopotential, with accuracies equivalent to about 5 cm in geoid height. Analysis of the eccentricity was also attempted, but did not yield useful results.

As no previous satellite analysed at 14th-order resonance has had an inclination near 40°, the results have proved to be valuable in determining individual 14th-order harmonics in the geopotential.

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